Behavioural Cournot competition

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CHAPTER 3

BIG IS BEAUTIFUL
Cournot competition, habit formation and exit

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1. Introduction

Starting from the seminal contributions of Beaver (1966) and Altman (1968) many studies into the (*ex post*) prediction of bankruptcies have appeared in the literature on Accounting and Finance (AF). The key issue in this literature is the identification of financial ratios that can predict corporate failure a few years before the actual date of bankruptcy. The results indicate that bankrupt firms are associated with financial ratios that started to deteriorate several years (in general, 1 to 5) before the year of failure (Foster (1986: Chapter 15)). For example, Zmijewski (1983) reports that in a sample of 72 bankrupt and 3,573 nonbankrupt firms over the 1972-1978 period the former showed a net income/net worth ratio of -.591 and the latter of 0.091 one year prior to bankruptcy. Another example is Hambrick and D'Aveni (1989). Hambrick and D'Aveni (1988: 10) report the results of an investigation into 57 large corporate failures during the period 1972-1982. They point out that "the bankrupts showed signs of relative weakness as early as ten years before failing. ... That these bankruptcies were culminations of often ten-year declines is compelling testimony to organizational inertia" (1988: 13 and 20). For example, in the five years prior to the date of bankruptcy (t) the series of the mean net income/assets ratio of bankrupt firms is –4.56 (t–5), –21.79 (t–4), –21.30 (t–3), –85.11 (t–2) and –107.89 (t–1).

Moreover, a few exceptional studies take account of nonfinancial predictors. Noteworthy are Altman, Haldeman and Narayanan (1977), Ohlson (1980), Zmijewski (1983), Keasey and Watson (1987) and Storey, Keasy, Watson and Wynarczyk (1987), which include a measure of size as predictor of corporate bankruptcy. Their results indicate that "size appears as an important predictor" (Ohlson (1980: 122)) as bankrupt firms are, on average, significantly smaller than nonbankrupt corporations. This result is supported by empirical research on entry into and exit from industries. McDonald (1986), Dunne, Roberts and Samuelson (1988 and 1989), Baden-Fuller (1989), Lieberman (1990) and Baldwin and Gorecki (1991) reported that the exit rate is, by and large, significantly higher among small and young firms. For example, Dunne, Roberts and Samuelson (1989: 689) report that "in summary, failure rates are lower for older plants ... and for larger plants" on the basis of patterns of failure statistics for over 200,000 plants that entered manufacturing in the U.S. in the period 1967-1977.

The bottom line is that the key finding of the bankruptcy prediction models is that firms start to accumulate losses many years before the actual date of exit. However, apart from critique of the methods of empirical analysis and sample selection (Zavgren (1983)), Foster (1986: 559) notes that "economic theory has played a small role in the development of univariate or multivariate distress prediction models." That is, the model testing is not theory-guided, but based on *ad hoc* arguments. As Rees (1990: 406) observes: "Most of the empirical studies ... are wide-ranging searches for any statistically reliable relationships between failure and accounting variables without the benefit of theoretical backing. As such they have been occasionally characterised as 'brute empiricism'". Therefore, "if the literature on distress/failure prediction is to progress further, then more explicit and formal modelling of the economic interests and decision processes of the firm's major stakeholders will probably have to be undertaken" (Keasey and Watson (1991: 100)). The current paper takes up this challenge.

So far, the scarce theoretical AF-contributions have been of three types. First, statistical ruin theory is applied to the issue of the determination of the risk of bankruptcy. For example, Vinso (1979) introduces initial reserves, fixed costs and expected profit flows to calculate a firm's risk of failure. Second, a few theoretical AF-models focus on explaining bankruptcy by modeling creditors' confidence in terms of catastrophe theory. Notably Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981) argue that poor financial performance may induce creditors to suddenly withdraw credit. So, this type of model is concerned with explaining the breakdown of chronically failing firms by interference of outside stakeholders. Third, Wadhwani (1986) and Simmons (1989) derive reduced-form equations from a profit-maximizing framework in which firms engaged in price-taking (perfect) competition take account of determinants such as (future) product prices, debt ratios, interest rates, inflation levels, money wages, bankruptcy costs, share prices and bankruptcy probabilities.

The theoretical modeling of financial distress is, however, still in its infancy. Particularly the role of strategic competition in explaining financial distress is, as yet, largely ignored. The isue of strategic competition is central to Industrial Organization (IO). For a long time IO has relatively ignored the issue of organizational failure. In the 1980s a countable number of analytical papers started to model exit decisions of firms in a competitive environment. The notable contributions are Jovanovic (1982), Lippman and Rumelt (1982), Ghemawat and Nalebuff (1985 and 1990), Fudenberg and Tirole (1986), Frank (1988), Reynolds (1988), Whinston (1988), Baden-Fuller (1989), Dixit (1989,1992), Jovanovic and Lach (1989), Fishman (1990), Londregan(1990) and Dierickx, Matutes and Neven (1991). All models start from the assumption that firms seek to maximize the discounted profit flow. The key issue in this literature concerns the question which (group of) firm(s) decides to exit first, where firms' heterogeneity is, by and large, reflected in cost or size features.

Without exception, the models assume quantity (Cournot) competition, either among n atomistic firms or two duopolists (with endogenous or exogenous profit levels). Competition is strategic, as individual profit levels crucially depend upon the (exit and output) decisions of rivals. Cost differences may originate from many sources, for example, efficiency of closing (Baden-Fuller (1989)), learning-by-doing (Jovanovic and Lach (1989)), scale economies (Ghemawat and Nalebuff (1985)) and talent (Frank (1988)). The results of the models are diverse, and appear to depend crucially upon the underlying assumptions. Two results are worth noting. First, the well-documented natural selection argument is supported if inefficient firms decide to exit first (Jovanovic (1982), Lippman and Rumelt (1982), Fudenberg and Tirole (1986), Frank (1988), and Jovanovic and Lach (1989)). Second, the contrary result holds if small and inefficient firms survive at the detriment of large and efficient rivals (Ghemawat and Nalebuff (1985,1990), Baden-Fuller (1989), Fishman (1990), Londregan (1990), and Dierickx, Natutes and Neven (1991)). This counterintuitive result is supported by Baden-Fuller's (1989) investigation of the steel castings industry in the U.K., being the only empirical IO-study on exit that takes account of profit performance. His finding is that, although a significant number of closures suffered from negative cash flow/sales ratios prior to exit, “the least profitable plants ... did not close” (1989: 956).

Except for Dixit (1989, 1992), the consequence of organizational failure in IO-models is just-in-time exit. This result is driven by the assumption of long-run backward induction. That is, firms are perfectly forward-looking: calculating the (expected) future stream of profit, taking account of the end-game equilibrium that results from strategic competition, they decide to exit at the moment profitability falls below zero. Although Dixit’s (1989) model describes a case where firms accept temporary losses, this result also follows from long-run calculations. The point is that Dixit (1989: 629) argues that firms take account of favourable demand developments in the future: “The firm knows that by remaining active it can avoid incurring [costs] for reentry should future developments turn favourable; therefore, it is willing to incur some current loss to preserve this option”. The focus of the IO-models on just-in-time exit (or in Dixit’s case: temporary losses) makes the results unable to explain the empirical findings in the AF-literature that more often than not firms accumulate losses before actually exiting the market. This state of the art is the result of IO-models’ reliance on assumptions of extreme rationality. Whinston (1988: 584, note 27) argues that “in fact, the results are, if anything, overly favourable about the possibility of prediction, since they rely heavily on long chains of backward inductive reasoning that are likely to be quite sensitive to even small amounts of irrationality”. After observing the same flaw Fishman (1990: 71) concludes that “this consideration should lead economists to proceed with caution before taking such results at face value”. The assumptions of extreme rationality are, moreover, reflected in the fact that all IO-models take firms to seek maximization of the future profit flow.

So, the AF-studies on financial distress clearly reveal that firms’ bankruptcies are, by and large, preceded by many years in which losses are accumulated. However, the formal modeling of the economics behind this result is still in its infancy, and the recent exit games in the IO-tradition fail to give in to the request for theoretical explanations by their exclusive focus on extreme rationality and just-in-time exit. This Chapter is a preliminary attempt to fill the gap by focusing on two strategies in particular: just-in-time (zero-profit) exit versus chronic failure (ongoing operation whilst accumulating losses). Van Witteloostuijn (1998) distinguishes four possible outcomes of the process of a firm’s downturn (measured in terms of profitability): immediate exit, turnaround success, flight from losses and chronic failure. He presents a framework that summarizes arguments from varying economic (IO) and organizational (OS) perspectives that have, for the most part, developed independently. His framework provides an overview of the literature on organizational decline, related to (internal and external) causes, (financial and nonfinancial) conditions, (shape and size) courses and (profits or losses) consequences. In this Chapter answers to two questions are investigated: under what conditions of competition and demand does either strategy (exit or chronic failure) occur?; and what features, in terms of efficiency and scale, characterize the firms that either exit or stay? Both questions are scrutinized by extending Vickers’ (1985) model of managerial economics by introducing cost asymmetries and habit formation. Like Vickers, Sklivias (1987) also considers a two-stage sequential game with cost symmetries, whereas Fershtman and Judd (1987) and Basu (1995) introduce cost asymmetries between competing firms. Concerning all these models, owners write an incentive contract for their managers in stage one of the game and then, in the second stage, managers play the Cournot or Bertand game. By backward induction the owners determine the incentive contract such that profitability is maximized, given the rival's incentive contract. This implies that the values of the weights (α or θ in these models) of the nonprofit parts of managers' objective functions (size or revenues) are determined and fixed (Nash equilibrium) by the owners in this two-stage game. So these models assume a high degree of rationality. However, we will examine all weight combinations (of the nonprofit parts of the objective functions) between rivals and will not restrict ourselves to the fixed weigths resulting from a two-stage sequential game. We consider the nonprofit-maximizing behaviour of managers (their "love for sales volume") as a habit, developed in time and such a habit can be part of the "blueprint" of the firm (Hannan and Freeman (1984)). The concept of habit formation has been applied to the explanation of preference changes in models of decision making on consumption (Pollak (1970) and Alessie and Kapteyn (1991)) and labour supply (Phlips (1978) and Vendrik (1992)). This Chapter introduces the notion of habit formation in the theory of the firm. This is explained in Section 3.2. Section 3.3 goes on to present the results of the model for the case where firms are facing symmetric cost conditions: that is, firms control equally efficient production technologies. Section 3.4 deals with cost asymmetries: both efficient and inefficient firms (may) operate in the market. Section 3.5 concludes the Chapter by offering an appraisal and conclusion.

2. A model

In accordance with the exit games in the IO-literature the model is Cournot: competition is in quantities. The prime deviation from standard Cournot, and so IO's exit games, is the introduction of sales (production volume) in the objective function of Cournot oligopolists. To be precise, the point of departure is a combination of Vickers' (1985) analysis of sales maximization in Cournot-Nash competition, Becker and Murphy's (1988) explanation of rational addiction through habit formation, and Akerlof's (1991) arguments in favour of myopic behaviour. These elements - sales maximization, habit formation and myopic behaviour - imply a departure from extreme rationality. Below the key assumptions are discussed in turn.

Vickers (1985) presents a modern version of the old theory of managerial economics. The key assumption in managerial economics is that firms (or, to be precise, managers) fail to maximize profits, but are driven by other motives, well-known examples being sales (Baumol (1953)), growth (Marris (1964)) and staff expenditures (Williamson (1964)). The common denominator of such nonprofit motives is that managers are assumed to favour the promotion of a measure of size: managers prefer to be head of large budgets, organizations or staffs rather than small ones. This hypothesis is confirmed by studies in public choice (Mueller (1989)), which, for example, indicate that bureaucrats maximize budgets (Niskanen (1971)). Similarly, theories of economic organization (Milgrom and Roberts (1992)), particularly principal-agent models, emphasize the nonprofit motives of nonowning managers (Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)). Note that size preference implies an asymmetry: managers like to grow, but dislike to retrench. Vickers (1985: 141) starts from the assumption that firms maximize in a period t

$$u_t = \Pi_t + \alpha x_t , \tag{3.1}$$

where α is a weight parameter, and u denotes utility, Π profit and x output or sales. So, equation (3.1) implies that "those who take the decisions in large firms are advanced by high sales rather than purely by profits" (Vickers (1985: 141)). The current model diverges form Vickers (1985) in three respects in order to fit more closely with the issues at hand: first, habit formation and myopic behaviour are included; second, cost asymmetries are introduced; and, third, firms are not subject to a zero-profit constraint. The bottom line is that these extensions permit an explicit focus on exit competition.

Becker and Murphy (1988) build upon the notion of rational habit formation (Stigler and Becker (1977) and Spinnewyn (1981)) in their explanation of addiction. As noted in the introduction, the concept of habit formation has been applied to the explanation of preference changes in models of decision making on consumption (Pollak (1970) and Alessie and Kapteyn (1991)) and labour supply (Phlips (1978) and Vendrik (1992)). The key argument is that people start to develop stronger preferences for consumption or working patterns over time if they get used to specific levels of consumption or numbers of working hours. A linear approximation of a habit formation function pertaining to Vickers' (1985) output or sales volume is

$$h_t = x_t + (1-\gamma)h_{t-1} \tag{3.2}$$

where γ is a depreciation factor and h denotes a habit parameter. If x is interpreted as consumption, equation (3.2) implies that “increases in past consumption raise current consumption” (Becker and Murphy (1988: 694)).

However, rational habit formation assumes that people in their current decisions take account of the future implications of developing a habit. As Akerlof (1991) notes, in practice “the standard assumption of rational, forward-looking, utility maximizing is violated” (1991: 1). This points to myopic habit formation. Recently, Akerlof (1991) lists arguments in favour of shortsightedness. For example, he notes that “the nonindependence of errors in decision making in the series of decisions can be explained with the concept from cognitive psychology of undue salience or vividness. For example, present benefits and costs may have undue salience relative to future costs and benefits” (1991: 1). One of his examples is organizational failure caused by escalating commitment (1991: 7-8), arguing that procrastination does not only occur in project initiation, but also in project termination. Note, moreover, that the standard assumption of Cournot-Nash conjectures implies a kind of myopic behaviour (to be precise, shortsighted expectations) as well. The three elements – sales (production) maximization, habit formation and myopic behaviour - give the essential assumption on firms’ decision making in the current model. To be precise, firms are assumed to maximize

$$u_t = \Pi_t + \alpha h_t \,, \tag{3.3}$$

which implies that firms maximize a utility function that is composed of profit and output, while output is subject to habit formation (h_t follows from equation (3.2)). That is, after a while firms increasingly prefer to be large; to paraphrase Becker and Murphy (1988: 694), increases in past size raise preference for current size.

Note that the interpretation of equation (3.3) from the perspective of studies on organizations (OS) is straightforward (Cameron, Sutton and Whetten (1988)). Evidence from OS-contributions clearly indicates that managers prefer volume. Particularly, studies of firms’ growth point out that “growth is frequently sought directly because it facilitates the internal management of an organization” (Whetten (1987: 30)). As far as habit formation is concerned, references to routinized behaviour abound in the OS-literature, notably the literature on forms of (managerial) inertia (Hannan and Freeman (1984) and Tushman, Newman and Romanelli (1986)). Note moreover that equation (3.3) implies that managerial inertia are asymmetric. Apart from the literature on corporate growth, Porter’s (1976) arguments on managerial exit barriers, for example, support this view: “Managerial exit barriers are characteristics of the company’s decision-making process which deter its management from making decisions to exit from a particular business even though they are justified on economic grounds” (1976: 23). Last but not least, the important role of myopic behaviour is stressed time and again in OS-contributions, an excellent example being Staw, Sandelands and Dutton (1981).

The model introduces strategic competition by assuming Cournot-Nash duopoly: two incumbent rivals, firm 1 and 2, compete over quantities by deciding on the output volume they bring to the market in period t. The model is completed by defining unit production cost c to be scale- and time-independent, and taking demand to be represented by the linear downward-sloping function

$$p_t = m_t - x_{1,t} - x_{2,t} \,, \tag{3.4}$$

where p denotes price, m is a size parameter, and x_1 and x_2 are the output volumes of firm 1 and firm 2, respectively. Noting that profit follows from $\Pi = (p-c)x$, substitution of the inverse demand function (3.4) and habit formation equation (3.2) into maximand (3.3) gives a firm's decision rule. That is, in a period $t + 1$ firm i decides to produce the output volume $x_{i,t+1}$ that maximizes

$$u_{t+1} = (m_{t+1} - x_{i,t+1} - x_{j,t} - c_i)x_{i,t+1} + \alpha_i[x_{i,t+1} + (1-\gamma_i)h_{i,t}], \tag{3.5}$$

where $i, j = 1, 2$ and $i \neq j$. Recall that the well-known Cournot-Nash assumption implies that firm i myopically expects firm j to sustain its period t's output volume in period $t + 1$: $x^e_{j,t+1} = x_{j,t}$, where e is an expectational operator. To focus on strategic competition only, assume for the time being that demand is stationary (m is time-independent) and cost is symmetric ($c_1 = c_2$).

Standard Cournot-Nash competition assumes pure profit maximizers, which follows from equation (3.1) or (3.5) by taking $\alpha_i = 0$. This gives the familiar Cournot-Nash duopoly equilibrium output (x^*) where $x^* = (m-c)/3$. Strict sales maximization without habit formation follows from $\alpha_i > 0$ and $\gamma_i = 1$, which reduces decision rule (3.5) to utility function (3.1). Then, Vickers (1985) shows that symmetric Nash equilibrium of the α-setting game (in a two-stage sequential game) gives output $x^* = 2(m-c)/5$. So, "compared with the case in which firms are managed by profit-maximizers, output per firm is higher, price is lower and profits are lower" (Vickers (1985: 142)). However, cases with negative profitability remain unexplored. On the one hand, if $m > c$, in both cases *neither* firm decides to exit, since *both* firms capture a positive profit in equilibrium. On the other hand, with $m < c$ *both* firms decide to undertake just-in-time exit. The outcome may be different if habit formation is introduced ($\alpha_i > 0$ and $0 < \gamma_i < 1$): that is, now α_i does not follow from highly rational decision making, but is the result of (fixed) managerial inertia. This allows us to consider a large set of (α_i, α_j) - combinations of the two rivals (satisfying nonnegative price restrictions) and reflect on the corresponding implications.

The basic model reflected in the equations (3.1)-(3.5) is developed below for two cases: Section 3.3 deals with cost symmetry ($c_1 = c_2$: Proposition 3.1), and Section 3.4 with cost asymmetry ($c_1 \neq c_2$: Proposition 3.5). In addition, spread over both sections attention is paid to five specific issues. Section 3.3 contains discussions of (i) disequilibrium and equilibrium profit (Proposition 3.2), (ii) speed of adjustment toward equilibrium (Proposition 3.2), and (iii) 'optimal' (that is: profit-maximizing) levels of habit formation (Proposition 3.4); Section 3.4 analyzes (iv) the n-firm case (Proposition 3.7); both sections include a discussion of (v) comparative statics of exit decisions (Propositions 3.3 and 3.6). When convenient, the results for both cases are compared in Section 3.4.

3. Cost symmetry

The first case assumes cost symmetry: $c_i = c_j$. By introducing $c_i = c_j = c$ in the basic model (3.1)-(3.5) stationary-state-equilibrium strategies can be calculated. The result is summarized in Proposition 3.1.

Proposition 3.1. A Cournot-Nash duopoly stationary-state-equilibrium with symmetric cost conditions ($c_1 = c_2$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically stable for $0 < \gamma_i, \gamma_j < 1$.

Proof. With $c_i = c_j = c$ maximization of utility function (3.5) gives a system of four difference equations:

(i) $x_{1,t+1} = -x_{2,t}/2 + (m - c + \alpha_1)/2$,
(ii) $h_{1,t+1} = (1-\gamma_1)h_{1,t} - x_{2,t}/2 + (m - c + \alpha_1)/2$,
(iii) $x_{2,t+1} = -x_{1,t}/2 + (m - c + \alpha_2)/2$, and
(iv) $h_{2,t+1} = (1-\gamma_2)h_{2,t} - x_{1,t}/2 + (m - c + \alpha_2)/2$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$, where $\underline{x}_t = [x_{1,t}, h_{1,t}, x_{2,t}, h_{2,t}]^{\mathrm{T}}$, $\underline{b} = [(m - c + \alpha_1)/2, (m - c + \alpha_1)/2, (m - c + \alpha_2)/2, (m - c + \alpha_2)/2]^{\mathrm{T}}$ and $A = \begin{bmatrix} 0 & 0 & -1/2 & 0 \\ 0 & 1-\gamma_1 & -1/2 & 0 \\ -1/2 & 0 & 0 & 0 \\ -1/2 & 0 & 0 & 1-\gamma_2 \end{bmatrix}$. The eigenvalues of matrix A are found by solving the equation $\det(A - \lambda I) = (1-\gamma_1-\lambda)(1-\gamma_2-\lambda)(\lambda+1/2)(\lambda-1/2) = 0$ with the sufficient and necessary conditions $\vee_i [|\lambda_i| < 1]$ for asymptotic stability. Hence, asymptotic stability occurs for $0 < \gamma_i < 1$, where i = 1, 2. From the equilibrium condition $\underline{x}^* = A\underline{x}^* + \underline{b}$ and so $(I - A)\underline{x}^* = \underline{b}$ follow the stationary-state-equilibrium outcomes (3.6), (3.7) and (3.8) below.

[End of proof]

If habit asymmetry is introduced ($\alpha_i \neq \alpha_j$), Cournot-Nash duopoly competition with habit formation gives firm *i*'s stationary-state-equilibrium output

$$x_i^* = (m - \alpha_j + 2\alpha_i - c)/3 \tag{3.6}$$

and stationary-state-equilibrium level of habit formation (h^*)

$$h_i^* = (m - \alpha_j + 2\alpha_i - c)/(3\gamma_i), \tag{3.7}$$

where i, j = 1, 2 and $i \neq j$. For brevity's sake henceforth the 'stationary-state-equilibrium' is, except in propositions and proofs, referred to as 'equilibrium'.

Equilibrium sales (3.6) support the intuition that firm *i*'s production exceeds firm *j*'s output if $\alpha_i > \alpha_j$. Firm *i*'s equilibrium profit (Π) follows from

$$\Pi^{\,i} = (m - \alpha_i - \alpha_j - c)(m - \alpha_j + 2\alpha_i - c)/9 \tag{3.8}$$

where *i, j* = 1, 2 and $i \neq j$. So, firm *i*'s profit falls below zero if $\alpha_i + \alpha_j > m - c$. Equation (3.6) confirms that symmetric pure profit maximization ($\alpha_i = \alpha_j = 0$) gives the symmetric Cournot-Nash equilibrium output $x^* = (m - c)/3$. Moreover the evident observation that firm *i*'s equilibrium output increases in its own habit formation (α_i) and decreases in its rival's preference for size (α_j), is supported, where the first force is twice as influential as the second ($+2\alpha_i$ versus $-\alpha_j$). Note that γ_i and γ_j have no impact on equilibrium volumes (but they do influence the habits h_i^* in equilibrium). Furthermore, note that the concept of habit formation doesn't influence equilibrium's stability. Proposition 3.2 deals with the speed at which the (supply) equilibrium is reached.

Proposition 3.2. Output volumes converge rapidly toward stationary-state-equilibrium values. The case where both firms decide to exit is reached immediately, for example. Therefore, the disequilibrium profit captured during the time of adjustment is negligible.

Proof.

We use the difference equations, concerning the firms' supplies (Proposition 3.1):

$$x_{1,t+1} = (m - c + \alpha_1)/2 - x_{2,t}/2\,,$$
$$x_{2,t+1} = (m - c + \alpha_2)/2 - x_{1,t}/2\,,$$

Using the initial conditions ($x_{1,0}, x_{2,0}$) we obtain the following solutions

$$x_{1,t} = (m - c - \alpha_2 + 2\alpha_1)/3 + (x_{1,0} - x_{2,0} + \alpha_2 - \alpha_1)(1/2)^{t+1} + \\ (-x_{1,0} - x_{2,0} + 2m/3 + \alpha_1/3 + \alpha_2/3 - 2c/3)(-1/2)^{t+1}$$

and

$$x_{2,t} = (m - c - \alpha_1 + 2\alpha_2)/3 + (x_{2,0} - x_{1,0} + \alpha_1 - \alpha_2)(1/2)^{t+1} + \\ (-x_{1,0} - x_{2,0} + 2m/3 + \alpha_1/3 + \alpha_2/3 - 2c/3)(-1/2)^{t+1}\ .$$

So both $x_{1,t}$ and $x_{2,t}$ converge to their stationary-state-equilibrium values rapidly. Take, for example, the case where both firms decide to exit. Then: $\alpha_1 \leq c - m$ and $\alpha_2 \leq c - m$, which implies that

$x_{1,1} = (m - c - x_{2,0} + \alpha_1)/2 \le -x_{2,0}/2 \le 0$ and
$x_{2,1} = (m - c - x_{1,0} + \alpha_2)/2 \le -x_{1,0}/2 \le 0$.

Hence, both firms simultaneously leave the market in period $t = 1$. Over $T + 1$ periods firm i's average profit per period ($i = 1, 2$), $\Pi_t^{\,i}$, follows from

$$\Pi_t^{\,i} = [1/(T+1)]\sum_{t=0}^{T}[(m - c - x_{1,t} - x_{2,t})x_{i,t}]\,.$$

Summing the sequences after substitution of the above expressions for $x_{1,t}$ and $x_{2,t}$ gives $\Pi_t^{\,i} = a_i(T) + \Pi^{\,i}$, where $a_i(T)$ indicates the disequilibrium profit during the adjustment phase and $\Pi^{\,i}$ is the stationary-state-equilibrium profit expressed in equation (3.8). The adjustment profit can be expressed as

$$a_i(T) = \frac{1}{T+1}\sum_{t=0}^{T}\left[E_1(\tfrac{1}{2})^{t+1} + E_2(-\tfrac{1}{2})^{t+1} + E_3(-\tfrac{1}{4})^{t+1} + E_4(\tfrac{1}{4})^{t+1}\right],$$

where the constants E_1, E_2, E_3 and E_4 depend on $x_{1,0}$, $x_{2,0}$, m, c, α_1 and α_2. Using summation of geometric sequences, one can easily observe that the adjustment profit goes to zero with increasing T. Since stationary-state-equilibrium values are reached rapidly, total profit can be approximated by ignoring the payoff during the period of adjustment, which gives equation (3.8).

[End of proof]

So, the period of adjustment after setting an arbitrary pair of (positive) starting values of output volumes, $x_{1,0}$ and $x_{2,0}$, is short. For the case where both firms exit in equilibrium this state of affairs is reached in the subsequent period ($t = 1$). Therefore, the value of total profit can be approximated by ignoring the adjustment payoff, which gives equation (3.8).

Starting from Propositions 3.1 and 3.2 a set of equilibria can be characterized. The equilibrium features (and, for that matter, adjustment dynamics) crucially depend on the precise values of α_1, α_2, m and c. The fact that prices cannot be negative, imposes three restrictions on the set of feasible equilibria. Demand function (3.4) indicates that nonnegative prices result if

$$x_1^* + x_2^* \le m \Leftrightarrow \alpha_1 + \alpha_2 \le m + 2c\,, \tag{3.9}$$
$$x_1^* \le m \Leftrightarrow 2\alpha_1 - \alpha_2 \le 2m + c\text{, and} \tag{3.10}$$
$$x_2^* \le m \Leftrightarrow 2\alpha_2 - \alpha_1 \le 2m + c\,. \tag{3.11}$$

Total exit implies that $x_i^* \le 0$. So, firm i leaves the market if

$$\alpha_j - 2\alpha_i \ge m - c\,, \tag{3.12}$$

where $i, j = 1, 2$ and $i \neq j$.

With the help of the equations (3.6) to (3.12) environmental regimes and competitive equilibria can be identified. For the sake of argument, four distinct cost-demand regimes (each being composed of many market periods t) are discussed. First, the benchmark regime (Figure 3.1) is the case where $c = 0 < m$. Second, in the next regime (Figure 3.2) the market can be served profitably by two standard Cournot-Nash duopolists: $0 < c < m$. Third, a border regime (Figure 3.3) occurs for $c = m$. Fourth, in the subsequent regime (Figure 3.4) demand has fallen below the level where any output volume can be sold profitably, either by duopolists or by a monopolist: $c > m$. The shift from Figure 3.1 to Figure 3.4 can be interpreted as being the result of a dramatic decline in demand in the sense that environmental conditions change from favourable to unfavourable. Compare, for example, Figures 3.2 and 3.4 for the cases where $c = 12$ and m drops from 15 (Figure 3.2) to 10 (Figure 3.4).

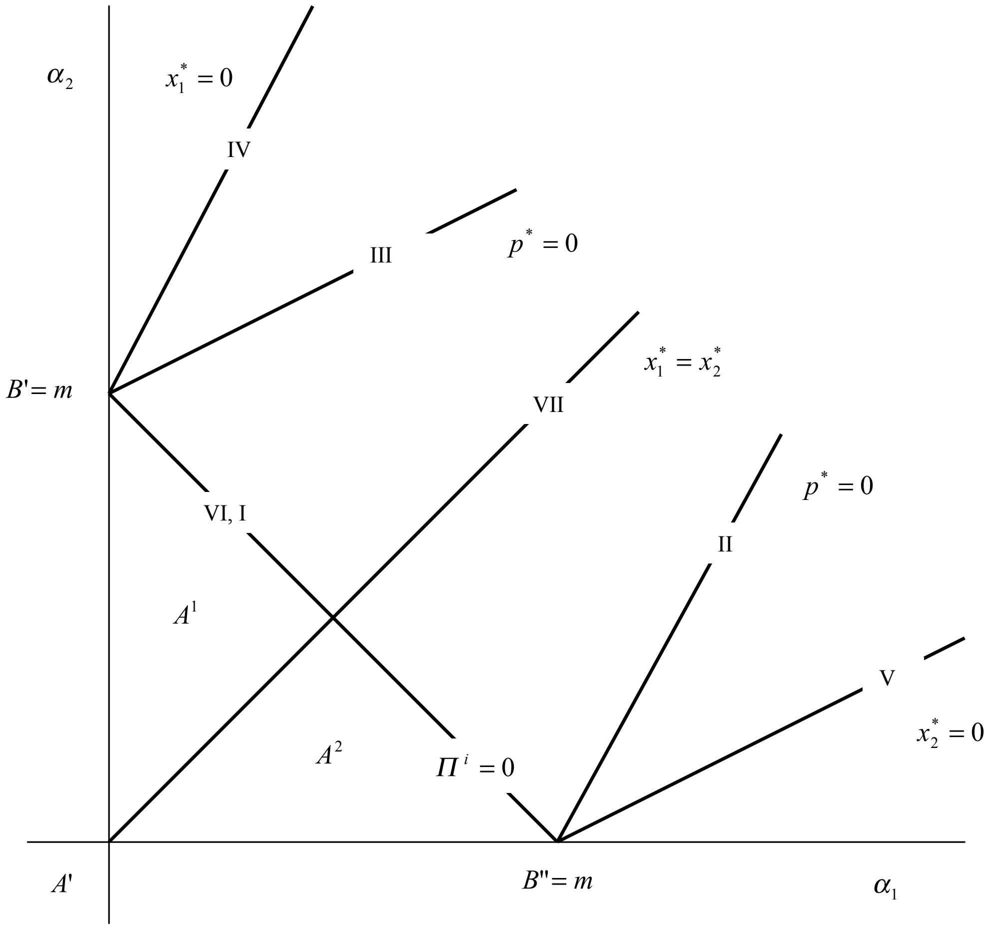

Fig. 3.1 Equilibrium outcomes for $c_1 = c_2 = 0 < m$

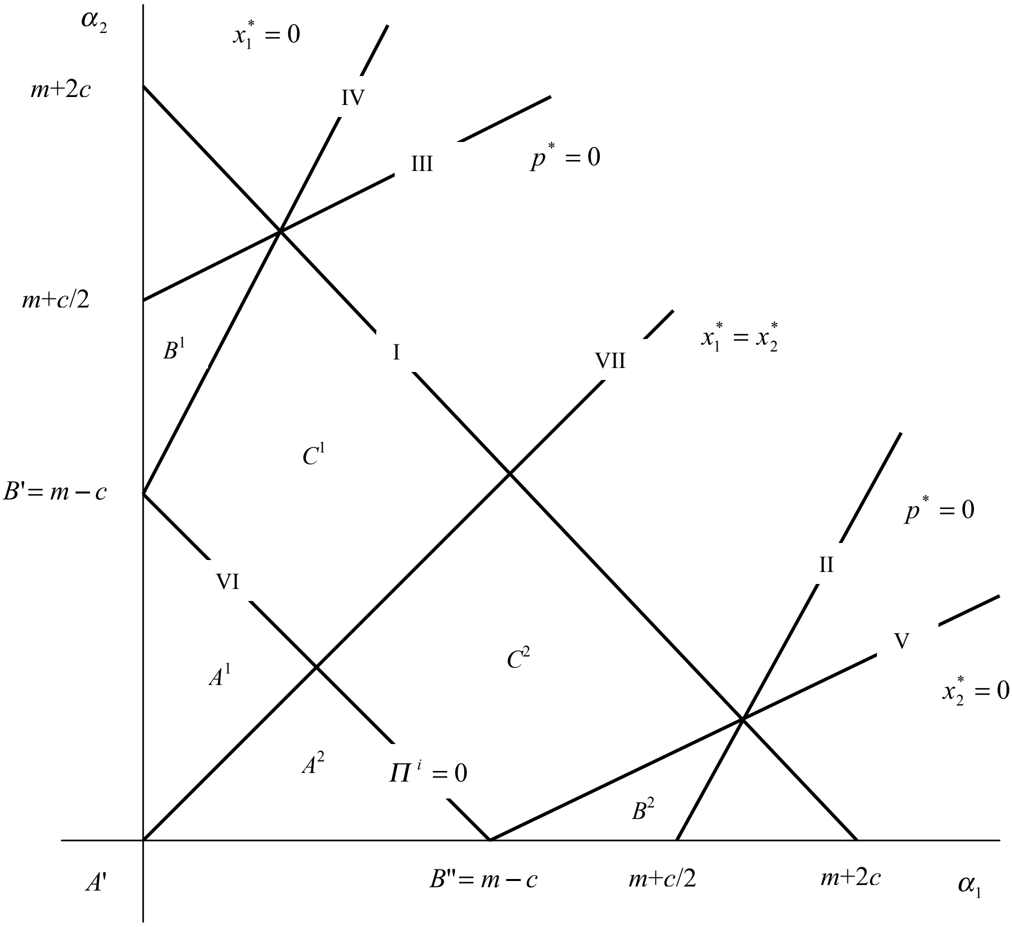

Fig. 3.2 Equilibrium outcomes for $0 < c_1 = c_2 = c < m$

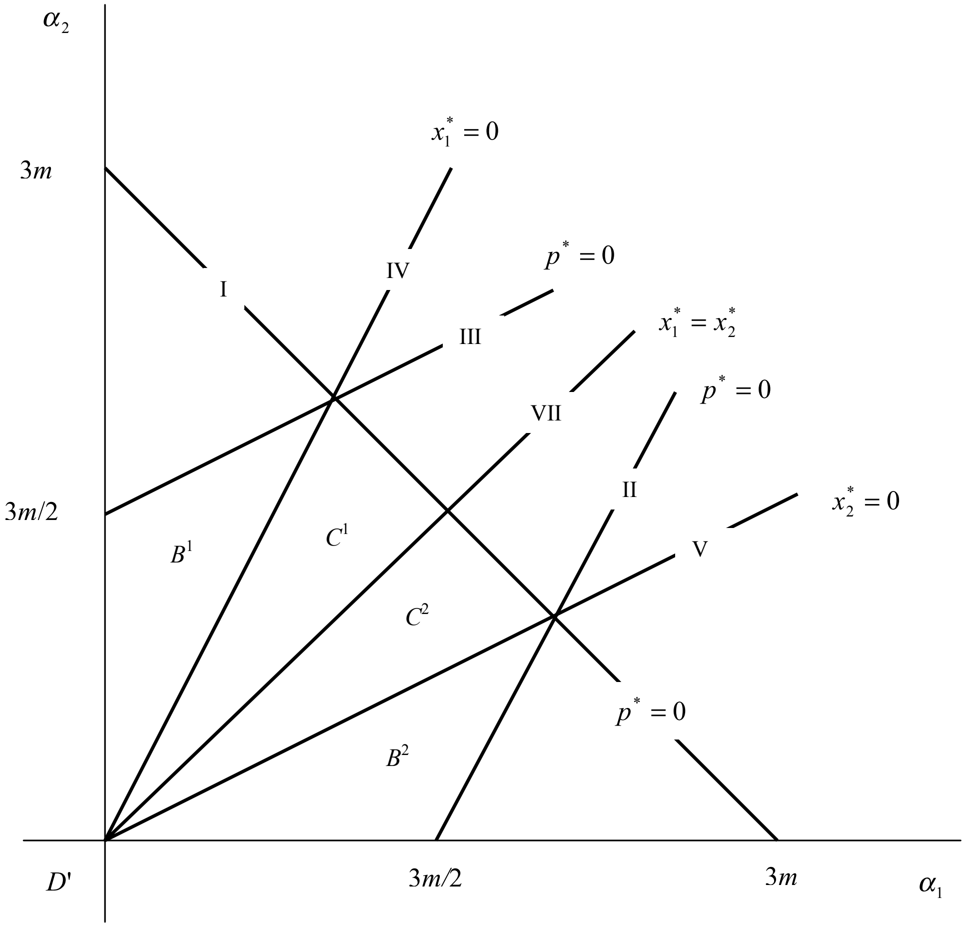

Fig. 3.3 Equilibrium outcomes for $c_1 = c_2 = c = m$

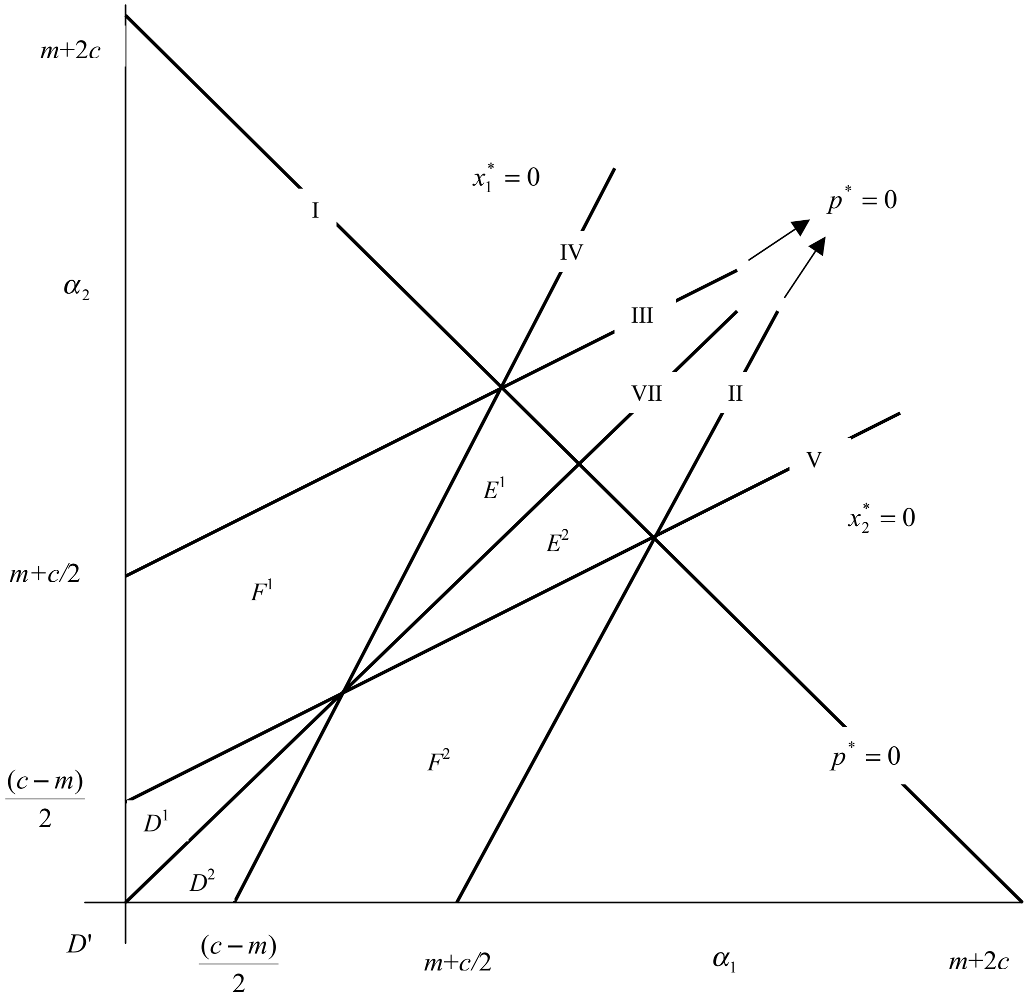

Fig. 3.4 Equilibrium outcomes for $c_1 = c_2 = c > m$

Beyond lines I, II and III price falls below zero (equations (3.9), (3.10) and (3.11), respectively). Lines IV and V depict firm 1's and firm 2's exit condition (equation (3.12): $\alpha_2 - 2\alpha_1 = m - c$ and $\alpha_1 - 2\alpha_2 = m - c$, respectively). Beyond lines VI profit starts dropping below zero (equation (3.8): $\alpha_1 + \alpha_2 = m - c$). At the right-hand side of lines VII firm 1's output exceeds firm 2's sales, whereas at the left-hand side of lines VII the opposite holds ($\alpha_1 > \alpha_2$ and $\alpha_2 > \alpha_1$, respectively). Figures 3.1-3.4 depict six specific equilibrium areas (*A-F*), apart from the border cases on the lines VI (zero profit) and VII (symmetric scale).

The six equilibrium outcomes can be briefly characterized as follows. In Figure 3.1 cost and demand conditions are favourable ($c = 0 < m$). Point *A'* is the standard Cournot-Nash equilibrium ($\alpha_1 = \alpha_2 = 0$). In equilibrium areas *A* both firms move away from standard Cournot-Nash by decreasing profit (though not below zero, given the condition that $c = 0 < m$) as a result of increasing habit formation ($\alpha_1, \alpha_2 > 0$), where firm j exceeds its rival i in size in equilibrium region A^i. On line VI the equilibrium takes the form of a zero-profit duopoly (with symmetric scale at the intersection point of the lines VI and VII, and with asymmetric size otherwise). In two limit cases the habit-motivated firm ($\alpha_i >> 0$) expels the profit-maximizing rival ($\alpha_j = 0$) from the market (points B' and B'') so that a zero-profit monopoly is reached. In Figure 3.2 cost conditions have deteriorated ($c > 0$), though not dramatically ($c < m$). Equilibrium areas A^i have decreased in size. In equilibrium areas B^i firm i is expelled from the market by its loss-making rival j, since firm j's managerial inertia dominates over firm i's preference for size ($\alpha_j >> \alpha_i$). Although market demand would enable two firms to produce profitably, in equilibrium areas C^i price falls below the level of average cost as a result of excessive market supply. Firm 1 is larger than firm 2 if $\alpha_1 > \alpha_2$ (C^2), firm 2 exceeds firm 1 in scale if $\alpha_1 < \alpha_2$ (C^1), and both firms are of equal size if $\alpha_1 = \alpha_2$ (line VII). In Figure 3.3 profit opportunities, even for a monopolist, are bound to disappear ($c = m$). Equilibrium areas A^i have finally disappeared. In equilibrium point D' both firms have decided to exit just in time. Neither firm is willing to accumulate losses for the sake of sustaining sales volume. Equilibrium point D' reflects a standard outcome of exit games in the IO-literature: $\alpha_1 = \alpha_2 = 0$ with simultaneous just-in-time exit. In Figure 3.4 the market is no longer viable, under whatever conditions ($c > m$), and equilibrium point D' has expanded to equilibrium area D: the profit motive dominates over habit formation. In equilibrium areas E^i, although the market is nonviable in terms of profitability, both firms stand the test of environmental decline (firm j being larger than rival i at either side of line VII). Given their preference for bigness, both firms are prepared to sacrifice profitability, even by accepting prices below the economic shut-down level ($p < c$). In equilibrium area F^i firm i gives in to deteriorating environmental circumstances, whilst firm j perseveres with operation. Contrary to firm i, firm j is willing to accept below average cost prices in order to be able to sustain sales volume.

So, from Figures 3.1 to 3.4 equilibrium areas appear and disappear, and grow and shrink. The results of this comparative statics can be summarized in Proposition 3.3.

Proposition 3.3. The (relative) size of the stationary-state-equilibrium areas where one or both firms decide to exit increases in c and decreases in m, with two notable exceptions: the size of the stationary-state-equilibrium area where only one firm leaves the market is independent from m for $0 \le c \le m$, and the stationary-state-equilibrium area where both firms stay in the market is independent from c for $c > m$.

Proof. The size of the admissible region is $\frac{1}{2}m^2 + \frac{1}{2}c^2 + 2mc$. Define three ratios of stationary-state-equilibrium areas: $R^1 = \dfrac{\text{One firm exits}}{\text{Both firms stay}}$; $R^2 = \dfrac{\text{Both firms exit}}{\text{Both firms stay}}$; and $R^3 = \dfrac{\text{Both firms exit}}{\text{One firm exits}}$. Note that $A = A^1 + A^2$, $B = B^1 + B^2$, $C = C^1 + C^2$, $E = E^1 + E^2$ and $F = F^1 + F^2$. By way of illustration, consider the following comparative statics. First, take the case where $0 \le c < m$. The size of the stationary-state-equilibrium areas A and C (where both firms stay in the market) is $\frac{1}{2}m^2 + 2mc - c^2$. Stationary-state-equilibrium area B (where one of both firms expels the rival from the market) is $\frac{3}{2}c^2$, which is independent from m. So, $R^1 = \frac{B}{A+C} = \frac{3c^2}{m^2+4mc-2c^2}$, indicating that for a fixed value of m the size of the exit area B increases with c. Note that $R^2 = R^3 = 0$. Second, both stationary-state-equilibrium areas are of equal size ($A+C=B$) if $c=m$: then $R^1=1$, and $R^2=R^3=0$. Third, take the case where $c>m$. Stationary-state-equilibrium area D (where both firms exit from the market) is $\frac{1}{2}(c-m)^2$. Stationary-state-equilibrium area E (where both firms stay in the market) is $\frac{3}{2}m^2$, which is independent from c. The size of stationary-state-equilibrium areas F (where only one firm leaves the market) is $\frac{3}{2}m(2c-m)$. So, with a fixed parameter m the ratios $R^1 = F/E$, $R^2 = D/E$ and $R^3 = D/F$ increase with c, implying an ((asymptotically) linear or quadratic) increase in the incidence of exit with increasing c: $R^1 = \frac{2c-m}{m}$, $R^2 = \frac{(c-m)^2}{3m^2}$; and $R^3 = \frac{(c-m)^2}{3m(2c-m)}$. Opposite results can be derived for the combination of variable m and fixed c.

[End of proof]

Of course, the result that the incidence of exit goes up if cost increases ($c\uparrow$) (and, for that matter, if demand decreases ($m\downarrow$)) is hardly surprising. Additionally, however, the model permits the calculation of (shifts in) absolute and relative sizes of the equilibrium areas. Figure 3.5 illustrates Proposition 3.3 for three cases: $\frac{\text{One firm exits}}{\text{Both firms stay}}$ ($R^1 = B/(A+C)$, or $R^1 = F/E$): Figure 3.5), $\frac{\text{Both firms exit}}{\text{Both firms stay}}$ ($R^2 = D/E$: Figure 3.6) and $\frac{\text{Both firms exit}}{\text{One firm exits}}$ ($R^3 = D/F$: Figure 3.7).

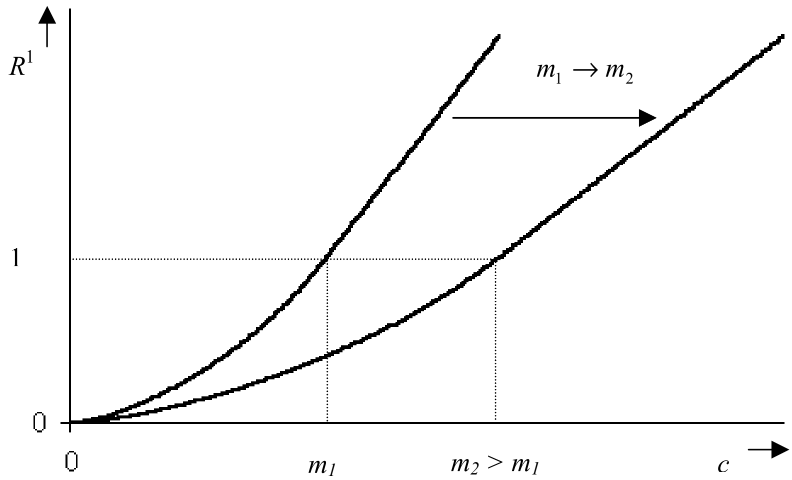

Fig. 3.5 Comparative statics for R^1: One firm exits / Both firms stay

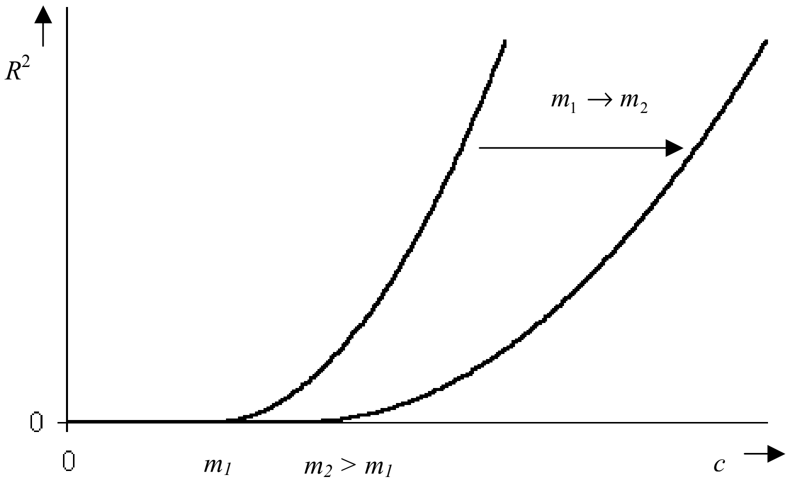

Fig. 3.6 Comparative statics for R^2: Both firms exit / Both firms stay

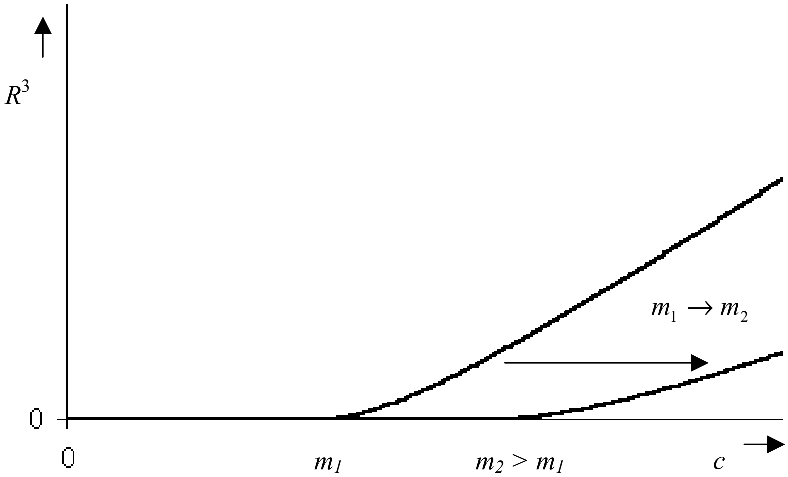

Fig. 3.7 Comparative statics for R^3: Both firms exit / One firm exits

Figures 3.5-3.7 depict the results for two cases of the demand parameter m: $m_1 < m_2$. Clearly, apart from the indicated exceptions, exit areas increase in c and decrease in m, either in a linear (R^1), quadratic (R^2) or asymptotically linear (R^3) way.

A final issue is related to Vickers' (1985) model, which assumes that firms *decide* on their preference for size (α) in an α-setting game. This paper relates to this issue by asking the question: what values of α_i maximize firm i's profit in equilibrium, given firm j's choice of α_j (we note that Fershtman and Judd (1987), Sklivias (1987) and

Basu (1995) also consider the choice of the weight attributed to revenues or, equivalently, sales volumes in a two-stage game; but we also consider the market size m , production costs c and habit formation symmetry). Proposition 3.4 indicates an answer to this question.

Proposition 3.4. With habit formation symmetry ($\alpha_i = \alpha_j = \alpha$) the profit-maximizing stationary-state-strategy for both firms is:

(i) negative habit formation ($\alpha < 0$) if $c < m$;
(ii) (zero-profit or standard, respectively) Cournot-Nash ($\alpha_i = \alpha_j = \alpha = 0$) if $c = m$ or $c < m$ and α restricted to be nonnegative; and
(iii) exit if $c > m$.

With habit formation asymmetry ($\alpha_i \neq \alpha_j$) the profit-maximizing stationary-state-reply of firm i to a fixed habit parameter α_j of firm j is:

(i) positive habit formation ($\frac{1}{4}(m - c - \alpha_j) > 0$) if $c < m$ and $\alpha_j < m - c$;
(ii) Cournot fringe follower ($\alpha_i = 0$) if $c < m$ and $\alpha_j = m - c$; and
(iii) exit otherwise.

Proof. First, take the case where $\alpha_i = \alpha_j = \alpha$. From equation (3.8) stationary-state-equilibrium profit $\Pi^i = \Pi^j = \Pi = \frac{1}{9}[-2\alpha^2 - (m-c)\alpha + (m-c)^2]$, which indicates a hill-shaped parabola of Π in α. Note that $\Pi = \frac{1}{9}(m-c)^2$ if $\alpha = 0$, and $\Pi = 0$ for $\alpha = \frac{1}{2}(m-c)$. Furthermore the fact that both firms stay in the market (positive production) leads to the condition $\alpha > -(m-c)$. The parabola shifts to the “South-East” with decreasing m. The parabola has a maximum at $\alpha = \frac{1}{4}(c-m)$, which is positive for $c > m$, zero for $m = c$ and negative for $c < m$. For $c > m$ stationary-state-equilibrium profit, Π, is negative, irrespective of the value of α.

Second, take the case where $\alpha_i \neq \alpha_j$. Assume that α_j is fixed and nonnegative. Equation (3.8) determines firm i’s stationary-state-equilibium profit, which is a hill-shaped parabola in α_i with one maximum at $\alpha_i = \frac{1}{4}(m - c - \alpha_j)$. The parabola shifts to the “South-West” if α_j increases. With $c > m$ or $c < m$ but $\alpha_j > m - c$ profit is negative, whatever value of α_i is considered (see also Figures 3.4 and 3.2). If $c < m$ but $\alpha_j > m - c$, the maximum is at $\alpha_i = 0$. In the case where both $c < m$ and $\alpha_j < m - c$, firm i’s profit is maximized at $\alpha_i = \frac{1}{4}(m - c - \alpha_j) > 0$.

[End of proof]

The intuition is illustrated in Figures 3.8 and 3.9 .

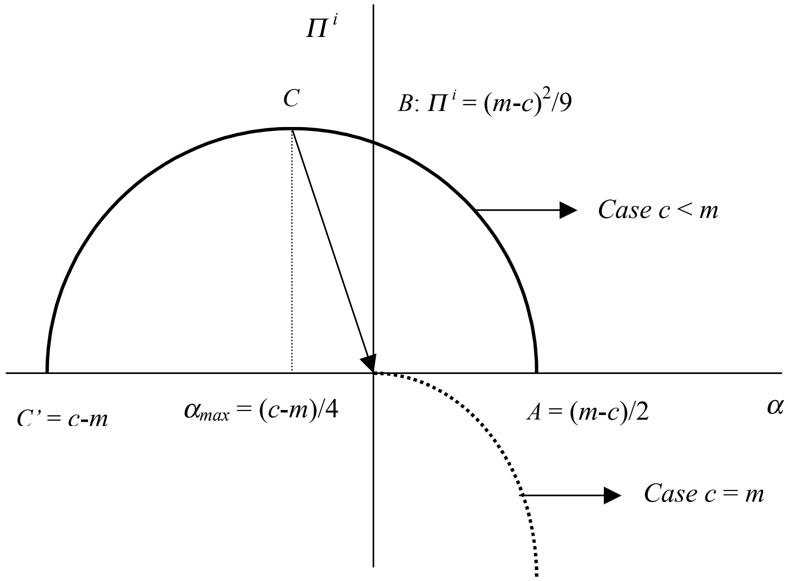

Fig. 3.8 The α-setting game for $\alpha_i = \alpha_j = \alpha$.

Figure 3.8 summarizes the implications for the case with habit symmetry. The curve $ABCC'$ depicts symmetric profit ($\Pi^1 = \Pi^2 = \Pi$) for symmetric habit formation ($\alpha_1 = \alpha_2 = \alpha$). The $ABCC'$-curve shifts to the "South-East" with decreasing demand parameter m. If $c = m$, the $ABCC'$-curve intersects the Π -axis at the parabola's maximum in (0,0), which indicates zero-profit Cournot-Nash behaviour as the profit-maximizing strategy. Exit is the profit-maximizing strategy for $c > m$, because profits always lie below zero. The interesting case is $c < m$. If α is restricted to be nonnegative, both firms maximize profit in the standard symmetric Cournot-Nash duopoly outcome by setting $\alpha_1 = \alpha_2 = \alpha = 0$ (point B), as the admissible equilibria are restricted to the first quadrant. That is, at point B profit is maximized by having no preference for sales volume, which gives the standard profit objective function. Moving from point B to A along the AB-line, profit declines with increases in sales preference. At point A a zero profit is earned, which resembles the perfectly competitive outcome of a standard Bertrand-Nash duopoly game. At point A α equals $\frac{1}{2}(m-c)$. The line AB is the right half of a parabola, which intersects the Π -axis at $\Pi = \frac{1}{9}(m-c)^2$ $(\alpha = 0)$. If α is allowed to be negative, the "left half" of parabola $ABCC'$, curve BCC', indicates a preference for smallness (that is, $\alpha < 0$), or: a negative utility of sales. At point C both firms maximize profit by restricting output, which resembles the collusive Cournot duopoly outcome ($\Pi^1 = \Pi^2 = \frac{1}{8}(m-c)^2$).

In Figure 3.9 the $EFGG'$-parabola depicts profit for α_i-choices, given a predetermined α_j. The $EFGG'$-curve shifts to the "South-West" if α_j increases, with its maximum at point H at (0,0) for $\alpha_j = m-c$. If $c < m$ and $\alpha_j < m-c$, the firm i benefits from positive habit formation up to a maximum at point F at $\alpha_i = \frac{1}{4}(m-c-\alpha_j)$. For $c < m$ and $\alpha_j = m-c$ this expression is maximized at point H

by setting $\alpha_i = 0$, which indicates Cournot fringe follower strategies by firm i in a leader-follower (Stackelberg) setting, where a small firm i ($\alpha_i = 0$) follows a large leader j ($\alpha_j >> 0$). If $c > m$ or $c < m$ but $\alpha_j > m - c$, exit is the profit-maximizing reply: the case with $c > m$ is trivial; with $c < m$ but $\alpha_j > m - c$ (see also Fig. 3.2) rival j has expanded such that firm i cannot operate profitably in the competitive fringe. Figure 3.9 supports, though in a different context, Dixit's (1992) observation that managerial inertia may well be optimal in the context of decision making in a dynamic environment.

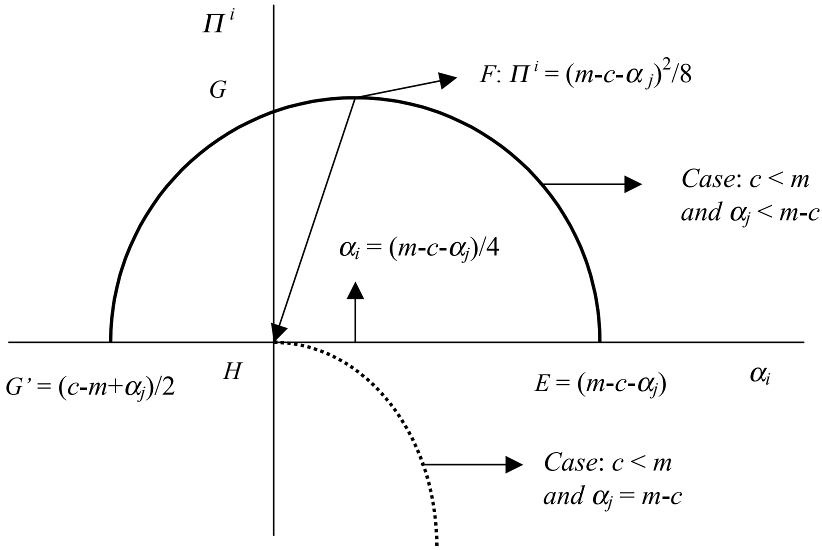

Fig. 3.9 The α-setting game for $\alpha_i \neq \alpha_j$.

4. Cost asymmetry

A popular explanation of organizational decline and exit is efficiency differentials: that is, $c_i \neq c_j$. The basic model (3.1)-(3.5) can now be re-analyzed, which gives Proposition 3.5.

Proposition 3.5. A Cournot-Nash duopoly stationary-state-equilibrium with asymmetric cost conditions ($c_i \neq c_j$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically stable for $0 < \gamma_i, \gamma_j < 1$.

Proof. With $c_i \neq c_j$ maximizing equation (3.5) gives a set of four difference equations:

(i) $x_{1,t+1} = -\frac{1}{2} x_{2,t} + \frac{1}{2}(m - c_1 + \alpha_1)$,
(ii) $h_{1,t+1} = (1-\gamma_1) h_{1,t} - \frac{1}{2} x_{2,t} + \frac{1}{2}(m - c_1 + \alpha_1)$,
(iii) $x_{2,t+1} = -\frac{1}{2} x_{1,t} + \frac{1}{2}(m - c_2 + \alpha_2)$, and
(iv) $h_{2,t+1} = (1-\gamma_2) h_{2,t} - \frac{1}{2} x_{1,t} + \frac{1}{2}(m - c_2 + \alpha_2)$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$ with

$$\underline{b} = [\tfrac{1}{2}(m - c_1 + \alpha_1), \tfrac{1}{2}(m - c_1 + \alpha_1), \tfrac{1}{2}(m - c_2 + \alpha_2), \tfrac{1}{2}(m - c_2 + \alpha_2)]^{\mathrm{T}}$$

and A as in the proof of Proposition 3.1. Equilibrium conditions $\underline{x}^* = A\underline{x}^* + \underline{b} \Leftrightarrow (I - A)\underline{x}^* = \underline{b}$ determine the stationary-state-equilibrium values (3.13), (3.14) and (3.20), which, from condition $0 < \gamma_i < 1$, are asymptotically stable.

[End of proof]

If $c_i \neq c_j$, firm i's equilibrium sales (3.6) transform into

$$x_i^* = \tfrac{1}{3}(m - \alpha_j + 2\alpha_i - 2c_i + c_j), \tag{3.13}$$

and firm i's equilibrium habit formation (3.7) changes into

$$h_i^* = \tfrac{1}{3}\frac{m - \alpha_j + 2\alpha_i - 2c_i + c_j}{\gamma_i}, \tag{3.14}$$

where $i, j = 1, 2$ and $i \neq j$. Note that equation (3.13) confirms the intuition that firm i's equilibrium output increases in firm j's cost level. Equilibrium sales (3.13) indicate that firm i's output exceeds firm j's production if

$$\alpha_i > \alpha_j + (c_i - c_j) . \tag{3.15}$$

Condition (3.15) implies that lower habit formation can be compensated by an efficiency advantage.

The equivalence of the nonnegative price condition (3.9) to (3.11) is

$$x_1^* + x_2^* \leq m \Leftrightarrow \alpha_1 + \alpha_2 \leq m + (c_1 + c_2)\,, \tag{3.16}$$
$$x_1^* \leq m \Leftrightarrow 2\alpha_1 - \alpha_2 \leq 2m - c_2 + 2c_1\text{, and} \tag{3.17}$$
$$x_2^* \leq m \Leftrightarrow 2\alpha_2 - \alpha_1 \leq 2m - c_1 + 2c_2\ . \tag{3.18}$$

Firm i's exit condition (3.12) shifts to

$$\alpha_j - 2\alpha_i \geq m - 2c_i + c_j\,, \tag{3.19}$$

where $i, j = 1, 2$ and $i \neq j$. Firm i's equilibrium profit (3.8) transforms into

$$\Pi^{\,i} = \tfrac{1}{9}(m - \alpha_i - \alpha_j - 2c_i + c_j)(m - \alpha_j + 2\alpha_i - 2c_i + c_j) \tag{3.20}$$

where $i, j = 1, 2$ and $i \neq j$. So, firm i's profit is negative for $\alpha_i + \alpha_j > m - 2c_i + c_j$.

By way of illustration, this section explores the implications of cost asymmetry for two cases: $0 < c_1$, $c_2 < m$ and $0 < c_1 < m < c_2$. For the sake of convenience, the assumption is that $c_1 < c_2$: so, firm 1 is the lowest-cost producer. If $0 < c_1$, $c_2 < m$, demand decline cannot explain exit, so that the focus is on the impact of strategic competition in combination with managerial inertia. Figures 3.1-3.4 are Figures 3.10-3.12's counterparts. The shift from Figures 3.1-3.4 to Figures 3.10-3.12 can, for example, be interpreted as being the result of a change in competitive conditions following an efficiency-enhancing innovation by firm 1. Two subcases can be discerned as to the size of firm 1's cost reduction (that is, $c_2 - c_1$): $m - 2c_2 + c_1 > 0$ (Figure 3.10) and $m - 2c_2 + c_1 < 0$ (Figure 3.11). For example, take the case where $m = 15$, $c_2 = 12$ and c_1 drops from 12 to 10 in the first subcase, and subsequently from 10 to 8 in the second subcase. The third case follows from $0 < c_1 < m < c_2$: firm 2, contrary to firm 1, cannot profitably operate in the market (Figure 3.12). This can be the result of, for example, a drop in demand from 15 to 10.

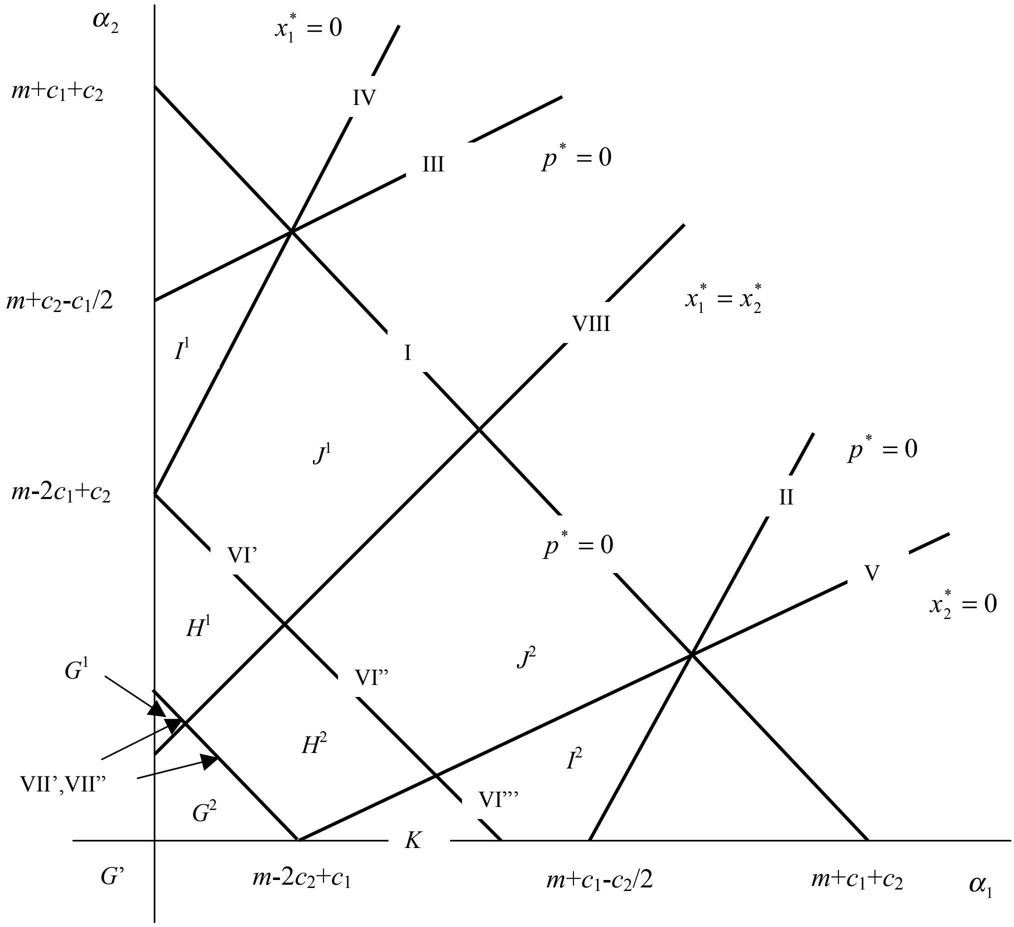

Fig. 3.10 Equilibrium outcomes for $0 < c_1 < c_2$ and $m - 2c_2 + c_1 > 0$

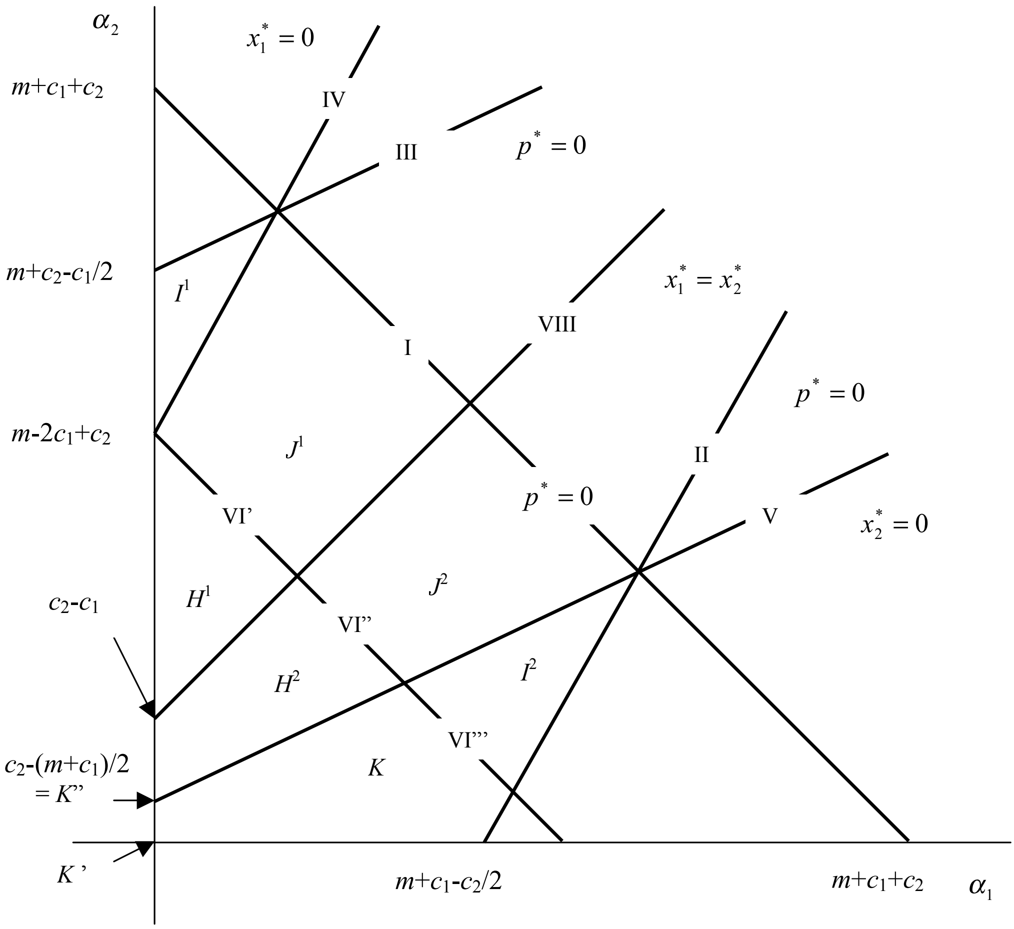

Fig. 3.11 Equilibrium outcomes for $0 < c_1 < c_2$ and $m - 2c_2 + c_1 < 0$

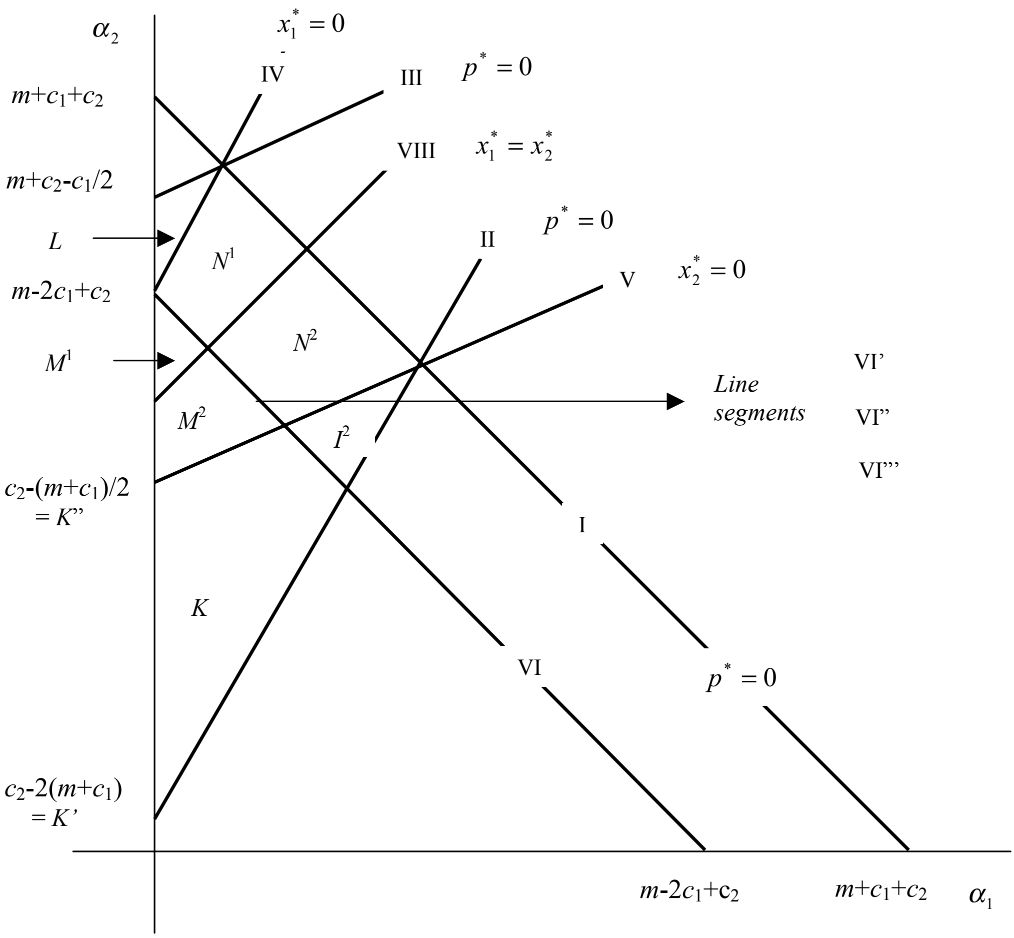

Fig. 3.12 Equilibrium outcomes for $c_1 < m < c_2$

The nonnegative price conditions determine the lines I, II and III (conditions (3.16) to (3.18), respectively). Lines IV and V represent firm 1's and firm 2's exit conditions, respectively (equation (3.19)). Lines VI'-VI''' and VII'-VII" depict firm 1's and firm 2's zero-profit points (equation (3.20)). Finally, at the right-hand side of line VIII firm 1's output exceeds firm 2's sales, whereas at the left-hand side of line VIII the opposite holds (equation (3.15)). For the sake of brevity, this symmetric scale case is not discussed explicitly below.

Relative to the case with cost symmetry, the introduction of cost asymmetries triggers results that are different in emphasis or nature. As far as shifts in emphasis are concerned, the fact that the firms' exit and profit conditions have changed in favour of the lowest-cost firm is worth noting. For example, the intuition is confirmed that the exit areas of the efficient and inefficient firm are relatively reduced (I^1 and L) and enlarged (I^2 plus K), respectively. Differences in nature are more interesting, however. First, the interpretation of equilibria may have to be changed. Most importantly, equilibrium areas I^1 and L imply that the inefficient firm 2 survives at the detriment of the lowest-cost rival 1, and at the left-hand side of line VIII in the equilibrium regions G^1, H^1, J^1, M^1 and N^1 and on the quilibrium lines VI' and VII' the highest-cost firm 2's production volume exceeds the sales of its efficient rival 1. In both cases the explanation is that the preference for bigness overcompensates the efficiency disadvantage in the motive scheme of firm 2, whereas the opposite occurs in firm 1's motivation structure. Note that equilibrium areas L, M^1 and N^1 even occur if $c_2 > m$ (Figure 3.12), implying that the inefficient rival 2 dominates over the efficient firm 1 even in an environment that is only viable for an efficient supplier!

Second, if c_1 is reduced to such an extent that $m - 2c_2 + c_1 < 0$ (Figures 3.11 and 3.12), the standard Cournot-Nash duopoly equilibrium G' (pure profit maximizers: $\alpha_1 = \alpha_2 = 0$) vanishes. The reason is that in this case firm 1's cost advantage is such that standard Cournot-Nash duopoly competition drives the profit of the inefficient firm below the economic shut-down point ($p < c_2$): firm 1 can exploit its efficiency advantage, and is able to operate as a natural monopoly. Third, two new equilibrium areas have emerged. In equilibrium regions H^i and M^i both firms have decided to stay in the market (firm j exceeding rival i in size). Cournot-Nash duopoly competition gives a price $c_1 < p < c_2$ (or $p = c_1 < c_2$ on lines VI' and VI''). So, firm 1 earns a positive profit (or zero on lines VI' and VI''), whilst firm 2 accumulates losses as a result of its efficiency disadvantage. Firm 2's habit of liking bigness overcompensates its profit motive. Condition (3.15) determines the firms' relative sizes. In equilibrium area K the lowest-cost firm 1 survives at the detriment of the inefficient rival 2. Firm 1 operates as a natural monopoly which exploits its cost advantage so as to capture a positive profit (or zero on line VI'''). Note that equilibrium line K'-K'' implies that $\alpha_1 = 0$, firm 1 being a standard, efficiency-protected monopoly. Firm 2's preference for large sales cannot compensate the negative profit so as to trigger a decision to stay.

In comparison with the cost-symmetric case (Proposition 3.3) the comparative statics of the exit game with efficiency differentials introduces an additional finding worth reporting. This finding is summarized in Proposition 3.6.

Proposition 3.6. For the case where $c_1 < c_2$, the (relative) size of the stationary-state-equilibrium areas where the inefficient firm 2 decides to exit increases (in a nonlinear way) in c_2 with one notable exception: the size of the exit stationary-state-equilibrium region of firm 2 is independent from c_2 for $c_2 \geq 2(m + c_1)$.

Proof. Define an additional ratio of stationary-state-equilibrium areas: $R^4 = \dfrac{\text{Firm 2 exits}}{\text{Firm 1 exits}}$. Distinguish three cases (for $c_1 < c_2$). For all cases stationary-state-equilibrium areas I^1 and L (where firm 1 decides to leave the market) are $\frac{3}{4}(c_1)^2$. This is different for the sum of stationary-state-equilibrium areas I^2 and K (where firm 2 leaves the market), and so for R^4.

(i) $c_2 < \frac{1}{2}(m + c_1)$: $I^2 + K = \frac{3}{4}(c_2)^2$; and $R^4 = \dfrac{(c_2)^2}{(c_1)^2}$.

(ii) $\frac{1}{2}(m + c_1) \leq c_2 < 2(m + c_1)$: $I^2 + K = \frac{3}{4}(c_2)^2 - \frac{1}{4}(2c_2 - c_1 - m)^2$; and

$$R^4 = \frac{(c_2)^2}{(c_1)^2} - \frac{1}{3}\frac{(2c_2 - c_1 - m)^2}{(c_1)^2}.$$

(iii) $c_2 \geq 2(m+c_1)$: $I^2 + K = \frac{3}{4}(m+c_1)^2$; and $R^4 = \frac{(m+c_1)^2}{(c_1)^2}$, which is independent from c_2.

[End of proof]

The result is illustrated in Figure 3.13, where $R^4 = \frac{\text{Firm 2 exits}}{\text{Firm 1 exits}} = \frac{I^2 + K}{I^1 + L}$.

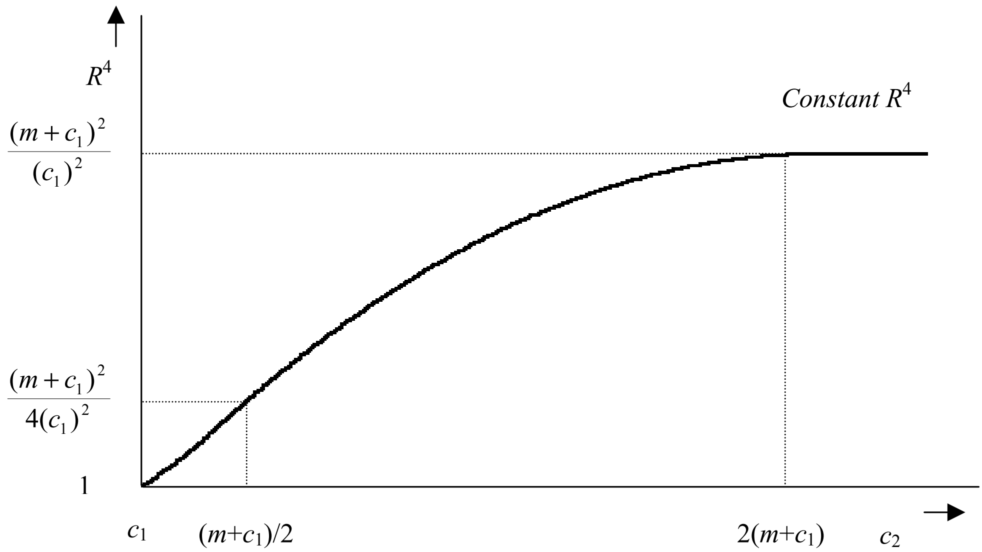

Fig. 3.13 Comparative statics for R^4: Firm 2 exits / Firm 1 exits

Figure 3.13 clearly depicts that R^4 remains constant beyond $c_2 \geq 2(m+c_1)$. For $c_2 < 2(m+c_1)$ the intuitive result is obtained that the ratio R^4, reflecting the relative incidence of firm 2's exit decision, increases (in a nonlinear way) in c_2.

The results for the duopoly model can easily be extended to the n-firm case. Proposition 3.7 summarizes the results of the n-firm Cournot-Nash competition game with habit formation and cost asymmetry.

Proposition 3.7. A Cournot-Nash oligopoly stationary-state-equilibrium with n firms, asymmetric cost conditions ($c_i \neq c_j$) and asymmetric habit formation ($\alpha_i \neq \alpha_j$) can be calculated, and is asymptotically instable for $n > 3$.

Proof. With n firms $p_t = m - \sum_{i=1}^{n} x_{i,t}$ (equation (3.4)). Maximizing the objective function (3.3) for period $t+1$, substituting $\Pi_{t+1}^{i} = x_{i,t+1}(m - x_{i,t+1} - c_i - \sum_{j \neq i} x_{j,t})$, and habit formation (3.2) for period $t+1$, gives the system of difference equations

$x_{i,t+1} = \frac{1}{2}(m - c_i + \alpha_i) - \frac{1}{2}\sum_{j \neq i} x_{j,t}$ and

$h_{i,t+1} = (1-\gamma_i)h_{i,t} + \frac{1}{2}(m - c_i + \alpha_i) - \frac{1}{2}\sum_{j \neq i} x_{j,t}$ for $i, j = 1,...,n$ and $i \neq j$.

In matrix form this is $\underline{x}_{t+1} = A\underline{x}_t + \underline{b}$, where

$\underline{x}_t = [x_{1,t}, h_{1,t}, x_{2,t}, h_{2,t}, ..., x_{n,t}, h_{n,t}]^{\mathrm{T}}$,

$\underline{b} = [\frac{1}{2}(m - c_1 + \alpha_1), \frac{1}{2}(m - c_1 + \alpha_1), \frac{1}{2}(m - c_2 + \alpha_2), \frac{1}{2}(m - c_2 + \alpha_2), ..., \frac{1}{2}(m - c_n + \alpha_n), \frac{1}{2}(m - c_n + \alpha_n)]^{\mathrm{T}}$,

and A the n-dimensional variant of matrix A in the proof of Proposition 3.1. Now the stationary-state-equilibrium volumes (21) and habit formations (22) can be calculated. The eigenvalues of matrix A are found by solving the equation

$$\det(A - \lambda I) = 0 \Leftrightarrow \det(A - \lambda I) = -\prod_{i=1}^{n}[(1-\gamma_i) - \lambda](\tfrac{1}{2} - \lambda)^{n-1}[\lambda + \tfrac{1}{2}(n-1)] = 0 .$$

Hence, noting that $\lambda_n = -\frac{1}{2}(n-1)$, the stationary-state-equilibrium outcomes are asymptotically unstable for $n > 3$. For $n = 3$ the eigenvalues are $\lambda_1 = 1 - \gamma_1$, $\lambda_2 = 1 - \gamma_2$, $\lambda_3 = 1 - \gamma_3$, $\lambda_4 = \lambda_5 = \frac{1}{2}$ and $\lambda_6 = -1$, which gives small upward and downward movements close to the stationary-state-equilibrium outcome. For $n > 3$ there exists one eigenvalue the absolute value of which exceeds 1. Now the fluctuations are limited by the nonnegative price restriction.

[End of proof]

Equilibrium volumes are

$$x_i^* = \frac{1}{(n+1)}(m - nc_i + \sum_{j \neq i} c_j + n\alpha_i - \sum_{j \neq i} \alpha_j) \tag{3.21}$$

and equilibrium habit formation is

$$h_i^* = \frac{1}{\gamma_i(n+1)}(m - nc_i + \sum_{j \neq i} c_j + n\alpha_i - \sum_{j \neq i} \alpha_j) \tag{3.22}$$

both for $i, j = 1,...,n$, $i \neq j$ and $n > 1$. Hence, if $\alpha_i = 0$ ($i = 1,...,n$) and $c_i = c_j = ... = c_n$, the standard Cournot-Nash outcome with n firms is reached. The counterparts of the nonnegative price conditions (3.16) to (3.18) are

$$\sum_{i=1}^{n} x_i^* \leq m \Leftrightarrow \sum_{i=1}^{n} \alpha_i \leq m + \sum_{i=1}^{n} c_i \text{ , and} \tag{3.23}$$

$$x_i^* \leq m \Leftrightarrow n\alpha_i - \sum_{j \neq i} \alpha_j \leq nm + nc_i - \sum_{j \neq i} c_j \tag{3.24}$$

for $i, j = 1,...,n$ and $i \neq j$. Now equilibrium profit can be calculated.

So, the n-firm case is a straightforward extension of the duopoly game implying a multiplication of the number of equilibrium regions without changing the qualitative features of the equilibrium outcomes. Model (3.21) to (3.24) defines an n-dimensional area being constrained by $n+1$ hyperplanes in the first quadrant ($\alpha_i \geq 0$ for $i = 1,...,n$). An important deviation from the duopoly case is however that for $n > 3$ asymptotic instability occurs. The model can be analyzed through computer simulation by supplementing the model with the condition that sales are not allowed to fall below zero. That is, in period t firm i's sales volume follows from

$$x^*_{i,t} = \max\left\{0, \left[-\tfrac{1}{2}\sum_{j \neq i} x^*_{j,t-1} + \tfrac{1}{2}(m + \alpha_i - c_i)\right]\right\} \tag{3.25}$$

for $i, j = 1,...,n$ and $i \neq j$.

5. Appraisal

The model reveals the precise conditions that underlie specific outcomes, ranging from standard Cournot-Nash duopoly competition over just-in-time exit to chronic failure, even by efficient firms. Essential in our model is that the weights attributed to size in the objective function, α_i, do (except from Proposition 3.4) not follow from highly rational decision making like in principal-agent models (such as Vickers (1985), Fershtman and Judd (1987), Sklivias (1987) and Basu (1995)), but are assumed to be the result of managerial inertia (habit formation). The analysis of all sorts of (α_1, α_2)-combinations leads to a variety of outcomes, such as an inefficient loss-making monopoly (region I^1 in Figures 3.10 and 3.11). On the one hand, as far as empirical regularities are concerned, the model outcomes support the two well-documented stylized facts referred to in the introduction. First, the observation that bankruptcy is negatively correlated to size is reflected in equilibrium areas B^i, F^i, I^i, K and L where the (larger) habit-motivated firm survives to the detriment of the (smaller) profit-motivated rival. Second, the empirical finding that, in many cases, firms accumulate losses before the actual date of bankruptcy appears in equilibrium areas B^i, C^i, E^i, F^i, H^i, I^i, J^i, L, M^i and N^i (ignoring the border cases on the line segments), where chronic failure is associated with pertaining negative profits for one or both firms.

On the other hand, from a theoretical angle the model reveals a broad set of outcomes and underlying causes through varying mixtures of cost (a)symmetries, demand turbulence, managerial inertia and strategic competition. Again, two remarks are worth making. First, IO-models concerning environmental decline, ignoring managerial inertia (so $\alpha_1 = \alpha_2 = 0$) are captured by the equilibrium points A' (cost-symmetric Cournot-Nash duopoly profit maximization), D' (duopoly just-in-time exit), G' (cost-asymmetric Cournot-Nash duopoly profit maximization) and K'-K'' (profit-maximizing efficient monopoly). Second, a key feature of this Chapter's model is the explanation of chronic failure by large, inefficient firms in equilibrium areas H^1, J^1, M^1 and N^1 (where the inefficient firm is larger than the lowest-cost rival) and, particularly, I^1 and L (where the inefficient firm has expelled the lowest-cost rival from the market). Table 3.1 summarizes the outcomes. For the sake of brevity, the cases with symmetric scale on the line segments VII (Figures 3.1-3.4, apart from the equilibrium point A') and VIII (Figure 3.10-3.12) are not included.

Table 3.1. *Equilibrium outcomes*

EQUILIBRIUM	INTERPRETATION	CONDITIONS
A'	Cost-symmetric profit-maximizing duopoly with symmetric scale facing favourable demand	$c_1 = c_2 = c$; $\alpha_1 = \alpha_2 = 0$; $\Pi^1 = \Pi^2 > 0$; $x_1^* = x_2^* > 0$; $0 \le c < m$
A^1,A^2	Cost-symmetric habit-motivated profit-making duopoly with asymmetric scale facing favourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $0 < \Pi^i < \Pi^j$; $0 < x_i^* < x_j^*$; $0 \le c < m$
VI	Cost-symmetric habit-motivated zero-profit duopoly with asymmetric scale facing favourable demand	$c_1 = c_2 = c$; $0 < \alpha_i < \alpha_j$; $\Pi^i = \Pi^j = 0$; $0 < x_i^* < x_j^*$; $0 \le c < m$

B', B''	Habit-motivated zero-profit monopoly facing favourable demand	$c_1 = c_2 = c;\ 0 < \alpha_i < \alpha_j;\ \Pi^i = \Pi^j = 0;$ $x_i^* = 0 < x_j^*;\ 0 \le c < m$
B^1, B^2	Habit-motivated loss-making monopoly facing favourable or neutral demand	$c_1 = c_2 = c;\ 0 < \alpha_i < \alpha_j;\ \Pi^j < \Pi^i = 0;$ $x_i^* = 0 < x_j^*;\ 0 < c \le m$
C^1, C^2	Cost-symmetric habit-motivated loss-making duopoly with asymmetric scale facing favourable or neutral demand	$c_1 = c_2 = c;\ 0 < \alpha_i < \alpha_j;\ \Pi^j < \Pi^i < 0;$ $0 < x_i^* < x_j^*;\ 0 < c \le m$
D'	Cost-symmetric profit-maximizing just-in-time exit with neutral or unfavourable demand	$c_1 = c_2 = c;\ \alpha_1 = \alpha_2 = 0;\ \Pi^1 = \Pi^2 = 0;$ $x_1^* = x_2^* = 0;\ c \ge m$
D	Cost-symmetric habit-motivated just-in-time exit with unfavourable demand	$c_1 = c_2 = c;\ \alpha_1,\ \alpha_2 > 0;\ \Pi^1 = \Pi^2 = 0;$ $x_1^* = x_2^* = 0;\ c > m$
E^1, E^2	Cost-symmetric habit-motivated loss-making duopoly with asymmetric scale facing unfavourable demand	$c_1 = c_2 = c;\ 0 < \alpha_i < \alpha_j;\ \Pi^j < \Pi^i < 0;$ $0 < x_i^* < x_j^*;\ c > m$
F^1, F^2	Habit-motivated loss-making monopoly facing unfavourable demand	$c_1 = c_2 = c;\ 0 < \alpha_i \ll \alpha_j;\ \Pi^j < \Pi^i = 0;$ $x_i^* = 0 < x_j^*;\ c > m$
G'	Cost-asymmetric profit-maximizing duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 > 0;\ \alpha_1 = \alpha_2 = 0;$ $0 < \Pi^2 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$
G^1	Cost-asymmetric habit-motivated profit-making duopoly facing favourable demand with inefficient leader	$m - 2c_2 + c_1 > 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^1,\ \Pi^2 > 0;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$
VII'	Cost-asymmetric habit-motivated duopoly facing favourable demand with inefficient zero-profit leader and efficient profit-making follower	$m - 2c_2 + c_1 > 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 = 0 < \Pi^1;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$
G^2	Cost-asymmetric habit-motivated but profit-making duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 > 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $0 < \Pi^2 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$
VII''	Cost-asymmetric habit-motivated duopoly facing favourable demand with efficient profit-making leader and inefficient zero-profit follower	$m - 2c_2 + c_1 > 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^2 = 0 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$
H^1	Cost-asymmetric habit-motivated duopoly facing favourable demand with inefficient loss-making leader and efficient profit-making follower	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 < 0 < \Pi^1;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$
VI'	Cost-asymmetric habit-motivated duopoly with inefficient loss-making leader facing (un)favourable demand and efficient zero-profit follower facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2;$ $\Pi^2 < \Pi^1 = 0;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$ (or $c_1 < m < c_2$)
H^2	Cost-asymmetric habit-motivated duopoly facing favourable demand with efficient profit-making leader and inefficient loss-making follower	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1);$ $\Pi^2 < 0 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$

VI''	Cost-asymmetric habit-motivated duopoly with efficient zero-profit leader facing favourable demand and loss-making follower facing (un)favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^2 < \Pi^1 = 0;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$ (or $c_1 < m < c_2$)
I^1	Inefficient habit-motivated loss-making monopoly facing (un)favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) << \alpha_2$; $\Pi^2 < \Pi^1 = 0;\ 0 = x_1^* < x_2^*;\ c_1 < c_2 < m$
I^2	Efficient habit-motivated loss-making monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1)$; $\Pi^1 < \Pi^2 = 0;\ 0 = x_2^* < x_1^*;\ c_1 < c_2 < m$
J^1	Cost-asymmetric habit-motivated loss-making duopoly facing favourable demand with inefficient leader	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 < \Pi^1 < 0;\ 0 < x_1^* < x_2^*;\ c_1 < c_2 < m$
J^2	Cost-asymmetric habit-motivated loss-making duopoly facing favourable demand with efficient leader	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^1,\ \Pi^2 < 0;\ 0 < x_2^* < x_1^*;\ c_1 < c_2 < m$
K'-K''	Efficient profit-maximizing monopoly facing favourable demand	$m - 2c_2 + c_1 < 0;\ \alpha_1 = 0 \le \alpha_2$; $\Pi^2 = 0 < \Pi^1;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
K	Efficient habit-motivated profit-making monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1)$; $\Pi^2 = 0 < \Pi^1;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
VI'''	Efficient habit-motivated zero-profit monopoly facing favourable demand	$m - 2c_2 + c_1 \gtrless 0;\ 0 < \alpha_2 << \alpha_1 + (c_2 - c_1)$; $\Pi^1 = \Pi^2 = 0;\ x_2^* = 0 < x_1^*;\ c_1 < m < c_2$
L	Inefficient habit-motivated loss-making monopoly facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) << \alpha_2$; $\Pi^2 < \Pi^1 = 0;\ x_1^* = 0 < x_2^*;\ c_1 < m < c_2$
M^1	Cost-asymmetric habit-motivated duopoly with inefficient loss-making leader facing unfavourable demand and efficient profit-making follower facing favourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 < 0 < \Pi^1;\ 0 < x_1^* < x_2^*;\ c_1 < m < c_2$
M^2	Cost-asymmetric habit-motivated duopoly with efficient profit-making leader facing favourable demand and inefficient loss-making follower facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^2 < 0 < \Pi^1;\ 0 < x_2^* < x_1^*;\ c_1 < m < c_2$
N^1	Cost-asymmetric habit-motivated loss-making duopoly with inefficient leader facing unfavourable demand and efficient follower facing favourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_1 + (c_2 - c_1) < \alpha_2$; $\Pi^2 < \Pi^1 < 0;\ 0 < x_1^* < x_2^*;\ c_1 < m < c_2$
N^2	Cost-asymmetric habit-motivated loss-making duopoly with efficient leader facing favourable demand and inefficient follower facing unfavourable demand	$m - 2c_2 + c_1 < 0;\ 0 < \alpha_2 < \alpha_1 + (c_2 - c_1)$; $\Pi^1,\ \Pi^2 < 0;\ 0 < x_2^* < x_1^*;\ c_1 < m < c_2$

This Chapter's model has, of course, its limitations. Table 3.1 reveals that the model does not cover two scenarios: temporary downturn, where a firm recovers after a temporary period of negative profit (Dixit (1989 and 1992)), and ultimate exit, where a firm leaves the market after a period of accumulated losses (Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981)). Van Witteloostuijn (1998) uses the terms “turnaround success” and “flight from losses” in his framework. Therefore, an immediate extension of the model could be in line with the theoretical exercitions from an AF-perspective (Ho and Saunders (1980) and Scapens, Ryan and Fletcher (1981)) by introducing the influence of institutional restrictions through modeling creditors’ confidence. This means that the firm’s profit constraint (or, to be precise, loss constraint) is endogenized rather than fixed to zero (IO-literature) of infinite (this Chapter).

Moreover, by way of illustration, three further extensions are worth mentioning. First, following Vickers (1985), the assumption of myopic behaviour may be relaxed. That is, habit formation becomes a strategic variable if managers (or other stakeholders, for that matter) are able to decide, at least to some extent, on routines formation by in advance taking account of the implications of specific decisional, organizational and ownership structures on the parameter α (in the models of Vickers (1985) and Fershtman and Judd (1987) owners write incentive contracts for their managers. Owners influence managers’ objective functions in such a way that profit is maximized, given the rival’s objective function). Proposition 3.4 reflects on the strategic choice of the weights α_i. Second, expansion (and shrinkage) may be assumed to be costly. In the well-established IO-tradition on decision making on capacity, expansion (or exit) requires investment (or loss) of sunk cost in building up (or breaking down) productive capacity (Tirole (1988)). Third, and related to the second, firms may be assumed to operate in a multimarket context, which has an impact on the nature and size of exit barriers (Van Wegberg and Van Witteloostuijn (1992) and Van Witteloostuijn and Van Wegberg (1992)). The key point is that the modeling framework in this paper permits the introduction of these and other extensions, and subsequently facilitates the analysis of the implications of newly introduced parameters.